Stability Analysis and Synchronization of Injection-Locked Semiconductor Lasers with Optical Feedback

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Chaos synchronization in semiconductor laser systems is a recent hot topic in nonlinear systems and is studied because of its potential to a communication technology\(^1\), \(^2\). In most systems, a bandwidth of the system is restricted by a relaxation frequency (internal modulation bandwidth) of a semiconductor laser. Thus the enhancement of the internal bandwidth of the laser is very important problem to pull up the system bandwidth. In this report, we propose an enhancement of the bandwidth of the chaos synchronization system by applying a strong injection-locking scheme\(^2\). Injection-locked semiconductor lasers with optical feedback behave chaotic oscillations in the enhanced bandwidth. Under certain conditions, the chaos synchronization in two bandwidth-enhanced lasers can be achieved.

Semiconductor lasers subject to optical feedback and optical injection are modeled in fig. 1. The model is described by rate equations as

\[
\frac{d}{dt} E(t) = \frac{1}{2} \left( 1 + i \alpha \right) G_N \left[ N(t) - N_{th} \right] E(t) + \gamma E(t - \tau) \exp(-i \omega \tau) + \gamma_{inj} E_{inj}(t) \exp(i \nu t),
\]

\[
\frac{d}{dt} N(t) = \frac{I}{e V} - \frac{N(t)}{\tau_s} - \left( G_N \left[ N(t) - N_{th} \right] + \frac{1}{\tau_p} \right) |E(t)|^2,
\]

where \(E(t)\) is complex envelope of laser electric field oscillating at angular frequency \(\omega\). And is expressed as \(E(t) = E_0(t) \exp\{i \phi(t)\}\). \(E_{inj}(t)\) is an injection laser field oscillating at \(\omega_{inj}\) and \(\nu\) is a detuning represented by \(\nu = \omega_{inj} - \omega\). \(\alpha\) is a linewidth enhancement factor and \(G_N\) is a differential gain constant. \(\gamma\) and \(\gamma_{inj}\) are feedback and injection rate expressed as \(\gamma = \gamma_{r} \) and \(\gamma_{inj} = \gamma_{inj} \eta\) (\(\eta\) is a coupling coefficient). \(N(t)\) is a carrier density in an active layer. \(I\) is an injection current, \(V\) is a volume of the active layer, \(N_{th}\) is a threshold carrier density, \(\tau_s\) is a carrier lifetime, and \(\tau_p\) is a photon lifetime. The used parameter values for following calculations are \(\alpha = 3\), \(G_N = 8.4 \times 10^{-13} \text{m}^3\text{s}^{-1}\), \(N_{th} = 2.018 \times 10^{24} \text{m}^{-3}\), \(\eta = 1.553 \times 10^{11} \text{s}^{-1}\), \(\tau_p = 1.927 \text{ps}\), and \(\tau_s = 2.04 \text{ns}\).

Stability of the laser is analyzed by a linear stability analysis. To apply the linear stability analysis to the eqs.(1) and (2), we assume a steady state solution of the laser as an injection-locked state (\(E(t) = E_S \exp\{i(\nu t + \phi_L)\}\)), where \(\phi_L\) represents a locking phase expressed

![Fig. 1. Mode of an injection-locked semiconductor lasers with optical feedback.](image-url)

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![Fig. 2. Calculated linear mode distributions. Circle, triangle, and square correspond to \(r_{inj} = 0.0, 0.3, \) and 0.5 respectively. Open and filled represent a feedback phase \(\phi = 0\) and \(\pi\), respectively.](image-url)

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as $\phi = \arcsin \left( \frac{-\nu}{\gamma \Omega (1+\alpha^2)} \right) \arctan \alpha$. An example of calculated linear modes are shown in fig.2. Damping rate $\Gamma$ of the highest linear mode is decreased for strong optical injection. Thus the laser keeps stable oscillation against optical feedback. In other words, the laser requires relatively strong optical feedback to enter into a chaotic state. Eigen-frequencies also shift to higher value as well as bandwidth enhancement in injection-locked semiconductor lasers. At strong optical injection, $\Gamma$ of the linear modes around the highest mode are increased and differences from the highest mode are decreased. On the other hand, $\Gamma$ of the modes that have low $\Omega$ are decreased. The linear modes are sensitive to the feedback phase and have large $\Gamma$ for destructive interference condition $\phi = \pi$ (where $\phi$ is a feedback phase denoted by $\phi = \omega \tau + \phi(t) - \phi(t-\tau)$) as lasers that have only an optical feedback. The eigen-frequency is enhanced as well as the case of $\phi = 0$ at $\phi = \pi$. These results indicate the laser includes typical characteristics both optical feedback (feedback phase sensitivity) and injection locking (bandwidth enhancement).

Next, we investigate the chaos synchronization of the lasers in master-slave configuration. A schematic diagram of the synchronization system is shown in fig.3. We considered an open-loop scheme. TL and RL are transmitter and receiver lasers with optical injection lasers (IL$_T$ and IL$_R$) for bandwidth enhancement, respectively. Numerical results are shown in fig.4. Fig.4(a) shows power spectra of the TL with or without optical injection by IL$_T$. We can see a clear shift of relaxation oscillation frequency from about 3 GHz (lower trace) to 8 GHz (upper trace). This result indicates that a communication bandwidth of the system is enhanced by a factor of 2.7. Fig.4(b) id a correlation plot between TL and RL. The receiver RL synchronizes to the TL. This synchronization state is complete type (anticipating) synchronization. We must note that the system is very sensitive to parameter mismatch. For example, the parameter range for the frequency detuning between TL and RL is about several tens of MHz. Thus we must pay much attention to the detuning between the transmitter and receiver lasers like reference 1.

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References
Abstract (35 words):

Stability and chaos synchronization are numerically studied in an injection-locked semiconductor laser subject to optical feedback. Enhancement of internal modulation bandwidth is recognized as well as a simple injection-locked laser. In master-slave configuration, two injection-locked lasers synchronize at chaotic state.