Relaxation Oscillation Enhancement and Coherence Collapse in Semiconductor Lasers with Optical Feedback

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The dynamic characteristics of a semiconductor laser with optical feedback are strongly dependent on the injection current and the reflectivity and position of the external feedback reflector. We investigated the relaxation oscillation enhancement and coherence collapse state of the laser oscillation based on the laser rate equations. It is well known that laser output power jumps with increase of the injection current due to external mode transition. But here for the first time we demonstrate the existence of a chaotic scenario within successive laser power jumps. The results calculated by numerical simulations based on the rate equations are compared with those of the experiments and good coincidence between them is found.

Key words: semiconductor laser, optical feedback, chaos, relaxation oscillation, coherence collapse

1. Introduction

The characteristics of semiconductor lasers with optical feedback have been attracted much attention since they provide an excellent physical model of the dynamics of nonlinear optical systems.\textsuperscript{1-14} They are also important from an application point of view such as for noise suppression in semiconductor lasers\textsuperscript{12} or cryptographic communications.\textsuperscript{13} Instability of laser oscillations including semiconductor lasers has been discussed from the early '70s, but it was in the '80s that the laser "noise" induced by the optical feedback effect was recognized as chaos. The dynamic behaviors in a semiconductor laser with optical feedback are influenced by three parameters in the system: the external feedback reflectivity, the external feedback length, and the injection current.\textsuperscript{12} In certain parameter regions, the laser oscillates rather stably, while it shows chaotic fluctuations in other regions.\textsuperscript{5,14} The important time scales or frequencies for the nonlinear system are those corresponding to the relaxation oscillation and external cavity modes, with the relaxation oscillation frequency playing an especially important role in the chaotic fluctuation of the laser output. The amplitude of the relaxation oscillation usually increases with increase of the external feedback reflectivity and the relaxation oscillation enhancement leads to coherence collapse of the laser oscillation.\textsuperscript{1-8,11} Recently, an intermittent route to chaos has been of particular interest in semiconductor lasers with optical feedback and is related to coherence collapse of a semiconductor laser. The phenomenon is known as low frequency fluctuations.\textsuperscript{15-19} In the presence of these fluctuations, the coherence of the laser is destroyed leading to a state of coherence collapse of the laser oscillation.

The dynamic behaviors of a semiconductor laser with optical feedback are not simple and are strongly dependent on the feedback reflectivity. According to the behaviors of the laser output, the dynamics can be characterized into five regimes (I ~ V) with increase of the feedback reflectivity.\textsuperscript{9} For the very small feedback regime I, the laser linewidth is increased or decreased depending on the phase of the light returned to the laser cavity. With increase of the feedback level, coherence collapse occurs in the laser output power, in which the laser linewidth is drastically broadened and the coherence length of the laser is much reduced. A very high feedback level (regime V) corresponds to stable laser operation.

Here, we are interested in regimes III and IV where the chaotic behaviors are dominant in the laser output power. In this paper, we devote particular attention to the relaxation oscillation and the route to coherence collapse in the compound cavity semiconductor laser. The relaxation oscillation plays an important role in the system. Chaotic oscillations from such a laser are strongly affected by the relaxation oscillation frequency.\textsuperscript{2,14} The dependence of the relaxation oscillation enhancement on the external reflectivity and the injection current is investigated. The relaxation oscillation enhancement and jumps of the laser output power depending on the injection current have been observed experimentally in earlier work by Lang and Kobayashi.\textsuperscript{20} In this paper, we have demonstrated for the first time the existence of the chaotic scenario within successive external mode jumps in the laser output power. Such phenomena have neither been known nor observed until now. The route to coherence collapse via relaxation oscillation enhancement and external cavity mode excitement is identified from numerical simulations based on the rate equations. We also conducted experiments; the numerical results are compared with these experiments and good coincidence between them is found.
2. Theoretical Background

Before the numerical simulations, we briefly summarize the theoretical background of a semiconductor laser with optical feedback. The dynamics of a single-mode semiconductor laser with weak or moderate feedback are written by the rate equations for the amplitude \( E(t) \) and the average carrier density \( N(t) \) in the active region as follows:

\[
\frac{dE(t)}{dt} = \frac{1}{2} \left\{ g(N(t)-N_0) - \frac{1}{\tau_p} \right\} E(t) + \frac{\kappa}{\tau_m} E(t-\tau) \cos \theta(t) + F_E(t), \tag{1}
\]

\[
\frac{d\phi(t)}{dt} = \frac{\alpha}{2} \left\{ g(N(t)-N_0) - \frac{1}{\tau_p} \right\} \frac{E(t)}{\tau_p} \sin \theta(t) + F_\phi(t), \tag{2}
\]

\[
\frac{dN(t)}{dt} = J - N(t) \frac{g(N(t)-N_0)}{\tau_n} |E(t)|^2 + F_N(t), \tag{3}
\]

with

\[
\theta(t) = \omega_0 t + \phi(t) - \phi(t-\tau). \tag{4}
\]

Here, \( g \) is the modal gain coefficient, \( N_0 \) is the carrier density at transparency, \( \alpha \) is the linewidth enhancement factor, and \( J \) is the injection current density. Also, \( \tau_p \) is the photon lifetime, \( \tau_c \) is the carrier lifetime, \( \tau = 2L/c \) is the external cavity round-trip time (\( L \) and \( c \) being the external cavity length and the speed of light in vacuum, respectively), \( \tau_m = 2\pi nL/c \) is the optical round trip-time within the laser cavity (\( n \) and \( L \) being the refractive index and cavity length of the laser medium, respectively), and \( \omega_0 \) is the angular frequency of the solitary laser. The important feedback parameter \( \kappa \) is written by

\[
\kappa = \frac{(1-\tau_0^2)}{\tau_0^2}, \tag{5}
\]

where \( \tau_0 \) is the amplitude reflectivity of the cavity and \( \tau \) is the amplitude reflectivity of the external reflector. The same reflectivities for both the front and rear facets of the laser are assumed. This is true for the laser used in the experiments and the analysis of the different reflectivities is a straightforward extension. The carrier density at the laser threshold is defined as \( N_{th} = N_0 + 1/g\tau_p \). The last terms in Eqs. (1)–(3) are the Langevin noises. These noises have delta-function-like correlations and their auto- and cross-correlation functions are given in the literature.

The laser oscillation angular frequency is proportional to the injection current and written by

\[
\omega_0 = \omega_c - \frac{\partial \omega_c}{\partial J} J, \tag{6}
\]

where \( \omega_c \) is the constant angular frequency at a certain injection current and \( \partial \omega_c/\partial J \) is the angular frequency conversion efficiency to the injection current. The relation between the solitary laser angular frequency \( \omega_0 \) and the laser oscillation frequency \( \omega_* \) for the steady state condition is given by

\[
\omega_* = \omega_c + C \sin \left( \omega_0 + \arctan \alpha \right), \tag{7}
\]

with

\[
C = \frac{\kappa}{\tau_m \sqrt{1 + \alpha^2}}. \tag{8}
\]

In Eq. (7), the laser oscillation frequency \( \omega_* \) generally has multiple solutions and the number of solutions depends on the parameter \( C \). The actual laser oscillation is determined both from the phase and gain conditions for the stable state. For larger value of \( C \), the laser becomes unstable and sometimes hops around among possible oscillation modes. Another important quantity of a semiconductor laser is the relaxation oscillation frequency \( f_R \)

\[
(f_R = \omega_R/2\pi, \text{where } \omega_R \text{ is the angular frequency}) \text{ and this is derived from a small perturbation analysis for the variables in the rate equations as} \tag{21}
\]

\[
\omega_R = \frac{gE_s^2}{\tau_p}, \tag{9}
\]

where \( E_s \) is the steady state value of the field amplitude and \( E_s^2 \) is proportional to the laser output power \( P \). For no external feedback, the relaxation oscillation of the laser output power is originated from the statistical Langevin noises involved in the rate equations in Eqs. (1)–(3). But it is much enhanced by the mixing of the original laser oscillation with the external feedback light.

3. Numerical Results

First, to look at how the relaxation oscillation and the external cavity mode relate to the chaos dynamics, the numerical simulations are made using the fourth order Runge-Kutta algorithm. The various parameter values of an AlGaAs CSP (channeled substrate planer) semiconductor laser with 780 nm wavelength used for the simulations are the same as those in Ref. 12) and can be compared with the following experiments. Figure 1 shows some examples of the laser output power for different external feedback reflections. The external cavity length is \( L = 15 \text{ cm} \) and the bias injection current is \( J = 1.3J_{th} \). For small external feedback, the relaxation oscillation is visible in the waveform, but it is not distinct. The relaxation oscillation increases with increase of the external feedback. For larger external feedback, the laser output power is unstable and shows chaotic oscillations.

Figure 2 are the optical spectra corresponding to Fig. 1. The spectrum is almost single for small external reflection, but the relaxation oscillation gradually grows as the external reflection increases. The relaxation oscillation frequency is 2.50 GHz which is coincident with the theoretically expected relaxation oscillation frequency for the solitary laser. With further increase of the external reflection, the external mode frequency is also excited in the spectrum (see Fig. 2(c)). This spectrum is slightly less than the frequency calculated from the external cavity.
length of $L=15$ cm (corresponding frequency of 1.0 GHz). The frequency induced by the mode hop in the actual laser is determined from the conditions in Eq. (7) and is not exactly equal to the frequency calculated from the external cavity length. It depends on the parameter value $C$; this value is always less than the frequency calculated from the external cavity length. For the large external feedback in Fig. 2(e), the spectrum is greatly broadened and this state corresponds to the coherence collapse of the laser oscillation. The route to coherence collapse via relaxation oscillation enhancement is well demonstrated in these figures.

Figure 3 shows a bifurcation diagram of the laser output power against the external feedback reflectivity at $L=15$ cm and $J=1.3I_{th}$. The Langevin noise terms are not included in this simulation to show the pure dynamics of the laser oscillation. For a small external feedback, the laser output power stays in a fixed state (actually, an almost constant output power with small relaxation oscillation fluctuation). Period-1 oscillation around the external reflectivity of 0.5% is originated from the relaxation oscillation. With further increase of external reflectivity, the output power becomes unstable and finally evolves into chaotic oscillation. The numerical simulations are conducted under various conditions, especially for different external reflectivities. It is found that the coherence is easily collapsed with a smaller external feedback reflectivity when the external cavity length elongates.

For a change of the injection current, mode hops corresponding to the external cavity modes occur in the laser output power. Figure 4(a) shows the laser output power versus the injection current within $I=1.30I_{th}$ to $I=1.35I_{th}$ at the external cavity length of $L=15$ cm and the external reflectivity of $r=1.0\%$, which corresponds to a
quasi-periodic state in Fig. 3. The laser output power jumps at every interval of the injection current of \(0.0065J_\text{th}\). The optical spectra are calculated from the simulated waveforms and the change in spectra is shown in Fig. 4(b). Around the injection current of \(J/J_\text{th}=1.32350\), a mode hop occurs in the output power, and thereafter the optical spectrum shows only small relaxation oscillation peaks. As the injection current increases, the relaxation oscillation is gradually enhanced and frequencies of the external mode and its higher modes merge to the spectrum. With further increase of the injection current, the output power becomes unstable and the spectrum is greatly broadened. The main peak becomes obscured, which corresponds to a coherence collapse state. Finally, the output power jumps to the next mode around \(J/J_\text{th}=1.33025\) and the laser recovers a stable oscillation. The similar stability, relaxation oscillation enhancement, and coherence collapse process are repeated with increase of the injection current. Though these laser output power jumps have been observed in previous work,\(^5\)\(^2\)\(^0\) it is demonstrated for the first time here that the chaotic scenario exists between successive intensity jumps. Figure 4(c) shows a bifurcation diagram of the output power corresponding to Fig. 4(a). After a mode jump, the laser oscillates with a relaxation oscillation frequency with small amplitude. The relaxation oscillation is gradually enhanced in the laser output power as the injection current increases and the laser becomes unstable with the mixed state of the relaxation oscillation and the external mode. Finally, the laser output power shows quasi-chaotic or chaotic oscillation which corresponds to a coherence collapse state. After the next mode jump, the laser recovers rather stable oscillation.

Fig. 3. Bifurcation diagram of the laser output power against the external feedback reflectivity. The parameter values are the same as those in Fig. 1, but the Langevin noises are not included.

Fig. 4. (a) Laser output power versus injection current at the external cavity length \(L=15\) cm and the external reflectivity of \(r=1.0\%\). (b) Optical spectra corresponding to (a). (c) Bifurcation diagram of the output power for the same injection current range as that in (a).
In this simulation, the output power is essentially resistance with the increase or decrease of the injection current. In an actual situation, the laser output power has a hysteresis with increase or decrease of the injection current. Figure 4(a) shows the laser output power versus the injection current at the external cavity length \( L = 15 \) cm and the external reflectivity \( R = 2\% \). The injection current was driven by a triangular waveform signal with a frequency of 200 Hz. In the numerical simulations in Fig. 4, we calculated the bifurcation diagram for the external cavity length \( L = 3 \) cm. Since the dynamics is very sensitive to the injection current, it is easier to observe spectra corresponding to each bifurcation at a somewhat shorter external cavity length because the span of the injection current due to the jumps in the L-I curve is wide enough to measure them with high accuracy. On the other hand, the coherence is easily collapsed at a somewhat longer external cavity length, so that we observed a coherence collapse state at a longer external cavity length in a later experiment. Optical spectra at the injection currents marked in Fig. 4(a) are investigated and displayed in Fig. 6(b). At the point 'a' followed by a mode jump, the laser oscillation stays stable. With an increase of the injection current from 'c' to 'd,' the relaxation oscillation grows. After that, the laser shows quasi-periodic oscillations and then unstable oscillations before returning to a stable oscillation at 'h.' A similar process is repeated as the injection

![Fig. 5. (a) Laser output power versus the injection current at the external cavity length \( L = 15 \) cm and the external reflectivity \( R = 2\% \). (b) Bifurcation diagram of the laser output power.](image)

In an actual situation, the laser output power has a hysteresis with the increase or decrease of the injection current. The injection current range are the same as those in Fig. 4(a). The laser output power shows chaotic oscillation throughout the range in the figure. This corresponds to the state of complete coherence collapse of the laser oscillation.

4. Experiments

Enhancement of the relaxation oscillation and chaotic bifurcations were examined in the experiments. The semiconductor laser used was a single mode CSP semiconductor laser (Hitachi HL7801E), which oscillated at a single wavelength of 780 nm and a maximum power of 5 mW. The threshold current of the laser was 43 mA, and the temperature was stabilized by an automatic temperature control circuit within a high accuracy of \( \pm 0.003^\circ \text{C} \). The laser output power was also monitored by a photodiode (PD) installed within the laser diode package. It is noted that the reflectivity employed in this experiment was the reflection fraction of the returned light intensity at the collimating lens to the intensity emitted from the lens. It was not exactly the fraction of the intensity fed back into the active region in the semiconductor laser. Due to the vignetting of the collimating lens (N.A. 0.25), half of the original laser power was lost. The fraction of the light returned to the active region of the semiconductor laser (thickness of the layer was 0.2 mm) was roughly estimated to be one over thirty due to the diffraction effect of the lens. Then, the total estimated fraction of the intensity actually returned to the active region was about one over several tens of the experimental ratio.

The dependence of the optical spectrum on the variations of the injection current was investigated in detail. Figure 6(a) shows the relation between the laser output power and the injection current at the external cavity length \( L = 3 \) cm and the external intensity reflectivity \( R = 1.5\% \). As mentioned, the laser output power has a hysteresis with increase or decrease of the injection current. The injection current was driven by a triangular waveform signal with a frequency of 200 Hz. In the numerical simulations in Fig. 4, we calculated the bifurcation diagram for the external cavity length \( L = 15 \) cm, but the external cavity length here was set to be \( L = 3 \) cm. Since the dynamics is very sensitive to the injection current, it is easier to observe spectra corresponding to each bifurcation at a somewhat shorter external cavity length because the span of the injection current due to the jumps in the L-I curve is wide enough to measure them with high accuracy. On the other hand, the coherence is easily collapsed at a somewhat longer external cavity length, so that we observed a coherence collapse state at a longer external cavity length in a later experiment. Optical spectra at the injection currents marked in Fig. 6(a) are investigated and displayed in Fig. 6(b). At the point 'a' followed by a mode jump, the laser oscillation stays stable. With an increase of the injection current from 'c' to 'd,' the relaxation oscillation grows. After that, the laser shows quasi-periodic oscillations and then unstable oscillations before returning to a stable oscillation at 'h.' A similar process is repeated as the injection...
current increases. The measured relaxation oscillation frequency (for example in spectrum 'd') is 4.5 GHz. The expected external cavity mode frequency in this case is about 5.0 GHz. Therefore, it is not easy to distinguish between external modes and the relaxation oscillation in spectra for quasi-periodic oscillations, but the difference between periodic and quasi-periodic oscillations is clear in the figure. The result well coincides with the simulations.

For a high external feedback reflection of $R=9.0\%$ at $L=15$ cm, mode jumps are not seen in the L-I characteristics and no peak is visible in the optical spectrum as shown in Figs. 7(a) and (b), respectively. This is the state of coherence collapse. However, the internal laser mode which is, for example, observed by a broad band spectrum analyzer is still single in this case. Therefore, we can treat a single mode oscillation model of the laser in the numerical simulations based on the rate equations to investigate the route to coherence collapse via relaxation oscillation enhancement. The experimental results are well compared with those in Fig. 5. For the onset of coherence collapse, the phenomena can be interpreted by a single mode model of the laser oscillation, but the multimode effects can sometimes be taken into account in an actual situation for stronger optical feedback.

5. Conclusions

We have investigated the relaxation oscillation enhancement in a semiconductor laser with optical feedback and the route to coherence collapse with increase of the external feedback reflectivity. It has also been shown that the relaxation oscillation enhancement is dependent on the external cavity length and the injection current. At moderate optical feedback, the laser output power increases with increase of the injection current and jumps at a certain current due to the external mode transition. After the mode jump, the laser oscillates rather stably with a small relaxation oscillation frequency, but the relaxation oscillation is gradually enhanced as injection current increases. Then, the oscillation corresponding to the external cavity mode merges with the laser output power. Finally, just before the next mode jump, the laser becomes unstable. At a high external reflectivity, we have observed a completely coherence collapsed state of laser oscillation. This collapse is dependent not only on the external feedback reflectivity but also on the injection current. Good coincidence between the numerical simulations based on the laser rate equations and the experiments was found. In actual applications of a semiconductor laser with optical feedback, care must be taken to account for the combination of the parameters of the external reflectivity, the external cavity length, and the bias injection current to assure stable oscillation of a semiconductor laser.

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References