Controlling Chaos of a Delayed Optical Bistable System

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High-dimensional chaos was controlled with the occasional proportional feedback technique in a delayed optical bistable system which consists of a laser diode interferometer with a delayed opto-electronic feedback loop. Both the experiment and the numerical simulation showed that a large number of periodic orbits can be stabilized by controlling the chaotic attractor. The transient state of the trajectory under control was demonstrated.

Key words: chaos, chaos control, laser diode interferometer, delay-differential model, optical bistable system

1. Introduction

Chaos is a universal phenomenon in systems with nonlinearity and delay. In practice, it is often desirable to avoid chaos in systems in order to obtain stable performance. Methods of obtaining complex periodic waveforms from a chaotic system have attracted considerable attention in recent research concerning chaos. One of the most active fields of study is chaos control. The concept was first introduced by Ott, Grebogi and Yorke who proposed an algorithm (OGY method) which applies appropriately estimated minute perturbations to an accessible system parameter to select and stabilize a certain periodic orbit. This idea indicates that a chaotic system can be turned into a system with multipurpose flexibility, meaning that one can obtain various desired orbits from only a simple system without dramatically modifying the configurations of the system. An alternative but related control method is the occasional proportional feedback (OPF) technique. Recently, we applied the OPF method to a delay-differential chaos system. In the experiment, a large number of periodic orbits were stabilized in a region that is chaotic without the control. Since the system in general involves high-dimensional dynamics for which no simple return map is available, the OPF method is especially effective for real-time control. We also showed that the synchronizing frequency, which is a key factor in performing the OPF control, can be readily estimated from the known parameters of the delay-differential system.

In this paper, we present a further study using this chaos control technique with both experimental and numerical analyses. The control mechanism was numerically simulated; from numerical analysis, one can determine an unstable orbit evolves into a stable periodic orbit when the control is activated. The numerical results agreed well with those of the experiments.

2. Experiments of the OPF Control in a Delay-Differential System

The chaotic system employed in the experiment was a delayed optical bistable system that we proposed previously, consisting of a laser-diode interferometer with an electronic delayed-feedback loop. The dynamics of the system involve a one-dimensional delay-differential equation, and the main parameters are the bias injection current of the light source and the delay time.

The OPF control circuit together with the chaotic system are schematically shown in Fig. 1. The laser diode output intensity is detected by a built-in photodiode and converted into a time-dependent electric signal $p(t)$. A variable offset is added to $p(t)$ to bring it within a window of adjustable width. When the waveform transits within the window, the window comparator outputs a pulse with a width coincident with the length of the transition. Meanwhile, a synchronizing signal is generated by a microcomputer. The frequency of the synchronizing signal (hereafter referred to as a sampling frequency) is related to the delay and relaxation times of the system which we will discuss in more detail later. When the synchronizing signal is coincident with the pulse from the window comparator, the sample and hold circuit are activated, acquiring the waveform voltage. A gate is employed to select only part of the sampled signal. The gate width is adjustable within the range from one microsecond to several milliseconds. An amplifier with variable gain, offset and polarity delivers the control signal $g(t)$ to the drive current of the second laser diode LD2 and, in turn, perturbs the injection current of the light source LD1 from its ambient value.

In a delay-differential system, the ratio $t_r/\tau$ dominates the effects of the discrete or continuous properties of the system, where $t_r$ is the actual delay time and $\tau$ represents the relaxation time of the system. The fundamental frequency of the delay-differential system, which corresponds to the imaginary part of the first unstable mode of the system, was known to be near $1/(2(t_r+\tau))$. When performing OPF control in a delay-differential system, this ratio is very important for determining the sampling frequency. Generally, the sampling frequency $f_s$ can be set as a fraction or a simple multiple of the fundamental frequency.

A large number of periodic orbits are stabilized for various delay times. One example is shown in Fig. 2 for a delay time of 0.12 ms. The free-running state in the
absence of the control signal was adjusted to a weak chaotic state as shown in Fig. 2(a). Fluctuation in the waveform was rather small. The bias injection current was set as 60.75 mA. The fundamental frequency for the period-2 orbit was computed to be 2.50 kHz for \( t_0 = 0.12 \) ms and \( \tau = 80 \mu s \). For the short delay time, the frequency can also be obtained from the peak position of the spectral distribution corresponding to the waveform in Fig. 2(a). The obtained value was 2.45 kHz which agrees with the calculated value within the experimental acceptance. Control can be easily realized by adjusting the synchronizing frequency, the offset, the window width, and the gate width. In the experiment, we successfully stabilized sub-harmonic periodic orbits up to the 10th order by setting the synchronizing frequency at rational fractions of the fundamental frequency. Figure 2(b) shows a stable waveform of the period-2 orbit.

The perturbations applied to the drive injection current of LD1 were calculated, and the values were less than 1.5 mA which is about 3% of the ambient value of the bias injection current of LD1. We found that the dominant factors in OPF control are the sampling frequency, the gate width, and the offset. Using OPF control, we stabilized various periodic orbits which never appeared in the bifurcation routes without the control. Even for those periodic orbits which were observed both in the bifurcation routes and under the control, the sustaining time of the waveform under the control was two-digits longer than that without the control. We also found the stability of the waveform to be greatly influenced by uncertain factors in the experiment such as the temperature fluctuation and other noise existing in the light sources.

3. Numerical Simulations of the OPF Control

To gain a better insight into the theoretical background of the OPF method, we constructed a numerical model for the OPF algorithm and performed simulations. Our purpose was to demonstrate that, by feeding back an appropriately selected part of the output signal to the control parameter, we can convert a chaotic attractor into various periodic orbits depending on the sampling frequency, the feedback gain, and the gate width. The numerical model of the OPF technique in the LDAI system is given by

\[
\frac{d\rho(t)}{dt} + \rho(t) = f[\rho(t-\tau), r] - c \sigma(t-t_0),
\]

\[g(t) = S[\rho(t)-\rho_0] \times \sum_{i} \left( 1 - \left( e^{-\lambda t} \right)^i \right),
\]

with

\[S(p-p_0) = (p-p_0) \times \int_a^b \frac{H(u-a) - H(u-b)}{J},
\]

and

\[\sigma(t) = H(u-a) - H(u-b).
\]

Here, \( H \) is a Heaviside function, \( c \) is the feedback gain, \( \rho_0 \) is an offset to the output signal, \( a \) and \( b \) are the upper and lower thresholds of the window comparator, \( T_s = 1/f_s \) is the sampling period, and \( \Delta = t_0 - t_1 \) is the gate width. The function \( f \) represents the system dynamics and is given by

\[f[\rho(t-\tau), r] = r - \mu \rho(t-\tau) \{ 1 - \cos(\omega \rho(t-\tau) - \phi_0) \}.
\]

where \( r, \mu, b, k, \) and \( \phi_0 \) are system parameters.

Figure 3 shows the results of the OPF control of the chaotic attractor. The free running state in Fig. 3(a) appears as a weak chaotic output. In Figs. 3(b) and 3(c), we show the stable period-6 and period-8 cycles obtained by
OPF control. The gate width was adjusted at 0.4 $t_\tau$ in both cases. The sampling period $T_s$ was 8.5 $t_\tau$ and 11.4 $t_\tau$ for (b) and (c), respectively. In the numerical simulations, we also extracted many unstable periodic orbits with almost the same period as the stabilized orbits. We found that stabilized orbits were usually very similar to one of the unstable periodic orbits, although the control signal might result in a small change in the waveform.

The evolution process of the trajectory under the control was also investigated. Figure 4 shows plots of a set of trajectories before and after the control. For all four figures, data within 50 $t_\tau$ intervals were employed to reconstruct the trajectory in a two-dimensional phase space. In Fig. 4(a), one can see the chaotic attractor in the absence of the control which corresponds to the time variation shown in Fig. 3(a). Figure 4(b) shows the trajectory immediately after commencement of the control. It seems that the trajectory undergoes a random walk before reaching a stable state. In Fig. 4(c), the trajectory after 100 $t_\tau$ is shown. The decrease in volume of the attractor in the phase space indicates convergence of the trajectory.

Finally, in Fig. 4(c), we obtained a stable periodic orbit. The transient time depends on the gate width and the feedback gain.

4. Summary

In conclusion, we applied the OPF technique to achieve dynamic control of high-dimensional chaos in a delay-differential system. We showed both experimentally and numerically that periodic orbits can be stabilized in a region which is originally chaotic in the absence of the control. The transient state of the trajectory under the control was investigated. Numerical analysis of the control algorithm and the results verified the effectiveness of the control. Our results suggest the applicability of the OPF technique in delay-differential systems.

Acknowledgments

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References